#### Fluids in Games

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### Introductory Bits

- General summary with some details
- Not a fluids expert
- Theory and examples

- Deformable
- Flowing
- Examples
  - Smoke
  - Fire
  - Water



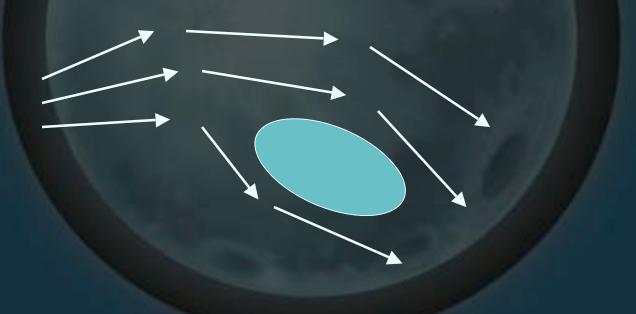




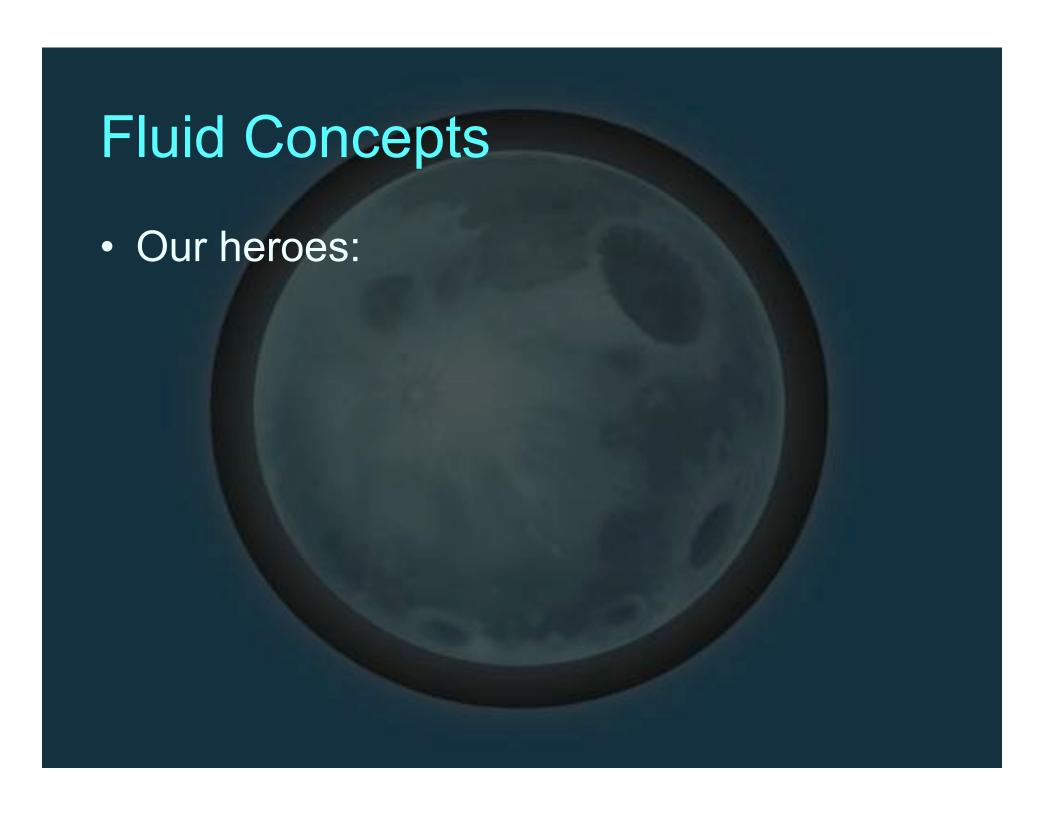


- Fluids have variable density
  - (Density field)

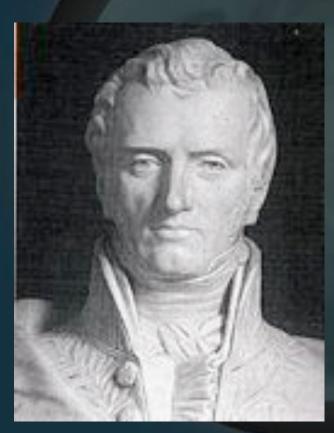
- Fluids "flow"
  - (Vector field)



- Need way to represent
  - Density (x)
  - Velocity (u)
  - Sometimes temperature



Our heroes:



Navier

Our heroes:



Navier

Stokes

Their creation:

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Their creation:

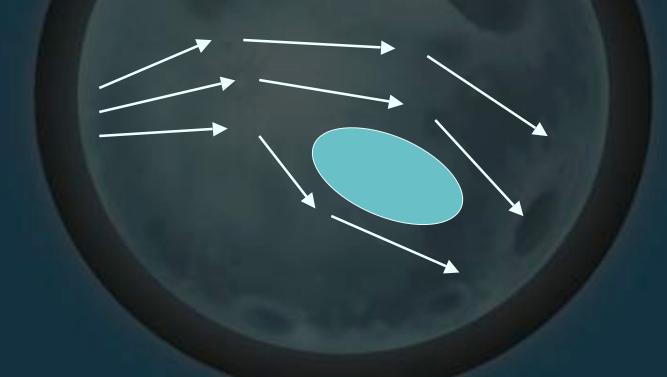
$$\nabla \cdot \mathbf{u} = 0$$

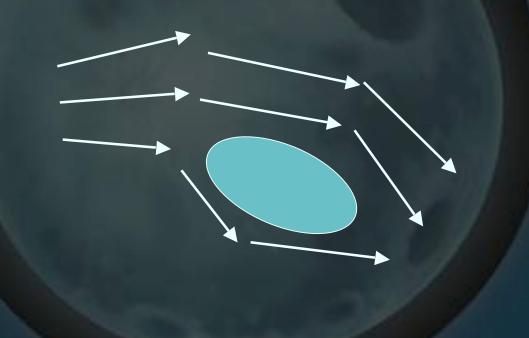
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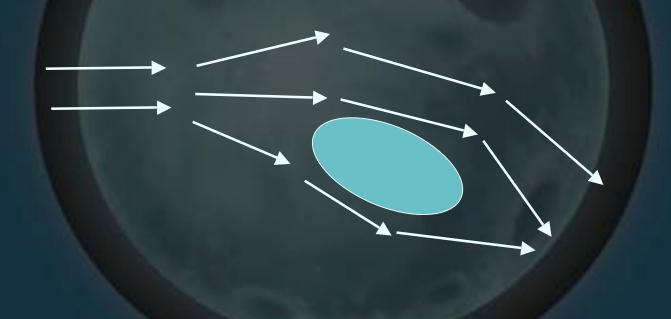
Their creation:

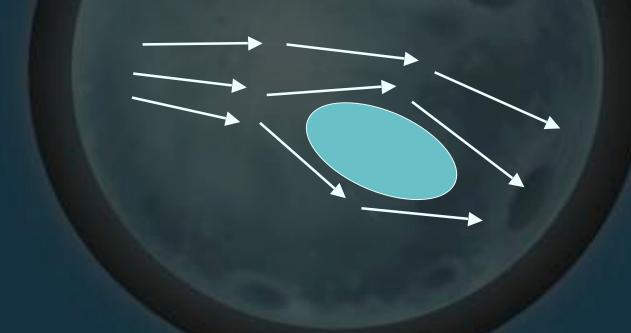
$$\nabla \cdot \mathbf{u} = 0$$

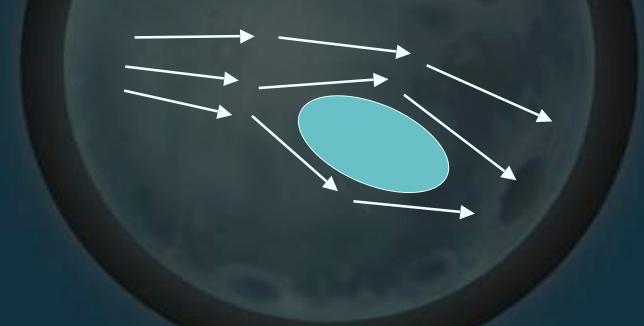
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$
SERIOUSLY --
WHAT DOES THIS MEAN?

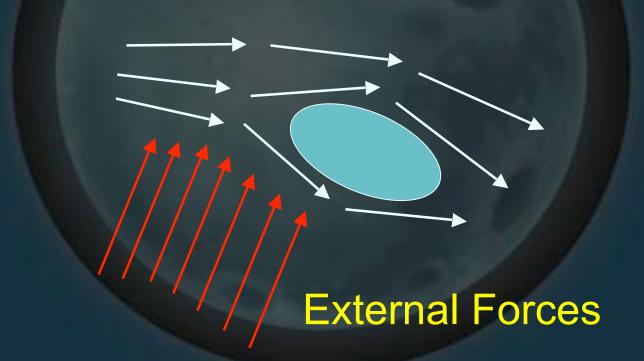


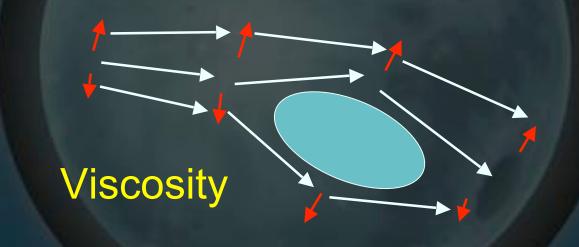


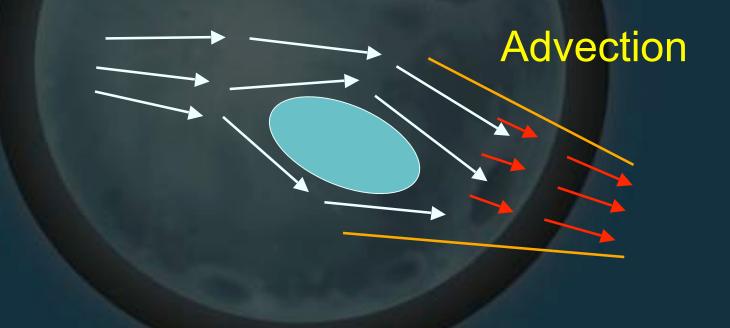












What affects it?

Pressure

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Brief notational diversion

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

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$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Gradient (vector along partial derivative)

Brief notational diversion

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Divergence (real derivative of vec. field)

Brief notational diversion

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \sqrt{2}\mathbf{u} + \mathbf{f}$$

Laplacian (divergence of gradient)

Brief notational diversion

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection operator (transport of flow)

Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Change in Velocity

Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} + \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

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Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

HOLD ON THERE BUCKO...

Back to Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

WHAT'S THIS ONE?

Back to Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

**Mass Conservation** 

In principle then, Navier-Stokes is...

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

In principle then, Navier-Stokes is...

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$
THE END!



But not really, of course

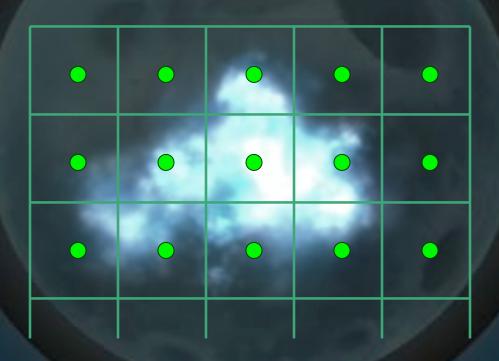


- But not really, of course
- Little tiny detail of implementation

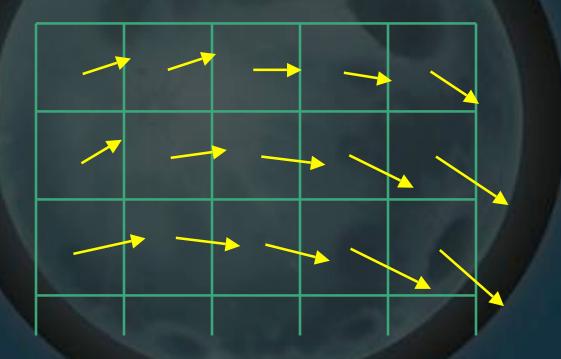
## Computational Fluid Types

- Grid-based/Eulerian (Stable Fluids)
- Particle-based/Lagrangian (Smoothed Particle Hydrodynamics)
- Surface-based (wave composition)

Store density, temp in grid centers



Velocity (flow) from centers as well



Could also do edges

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

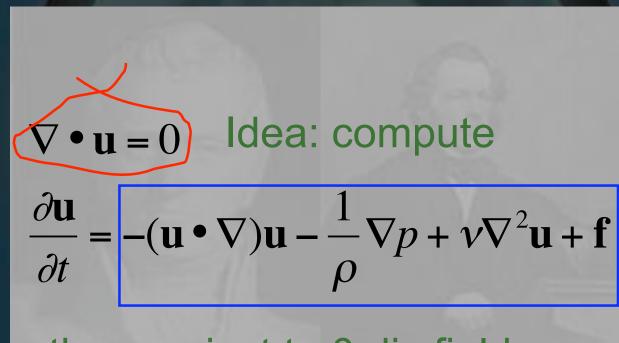
 Jos Stam devised stable approximation: "Stable Fluids", SIGGRAPH '99

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{u} \cdot \nabla \mathbf{u} = 0$$
 Idea: compute
$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

How do we use this?



then project to 0 div field

How do we use this?

End up with

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

How do we use this?

Sparse linear system

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla^2 \mathbf{u} + \mathbf{f})$$

How do we use this?

Sparse linear system

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$

How do we use this?

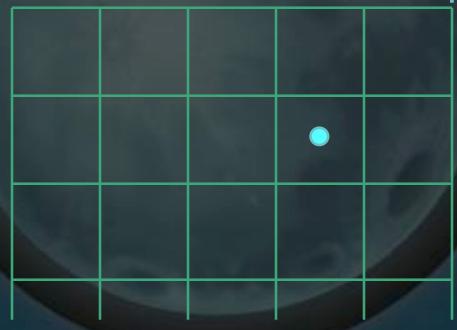
Non-linear... ugh

$$\frac{\partial \mathbf{u}}{\partial t} = \mathbf{P}(-(\mathbf{u} \cdot \nabla)\mathbf{u} + \nu \nabla^2 \mathbf{u} + \mathbf{f})$$



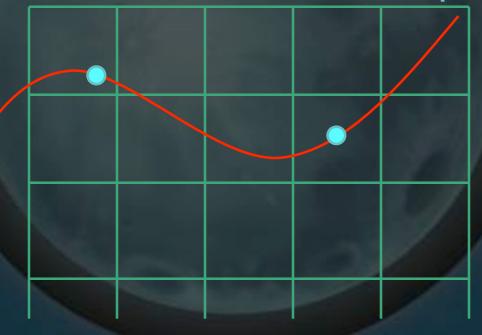
Updating advection

General idea: look at current position



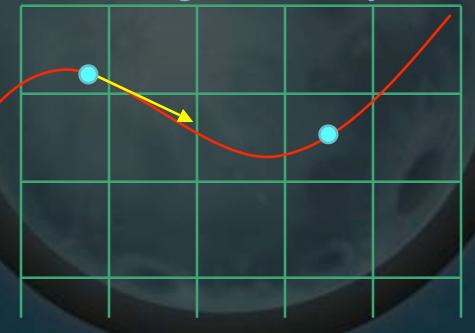
Updating advection

General idea: follow flow to prev. position



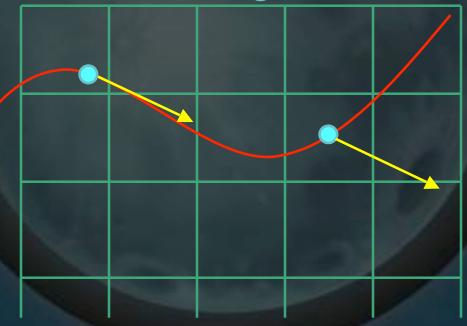
Updating advection

General idea: get velocity there



Updating advection

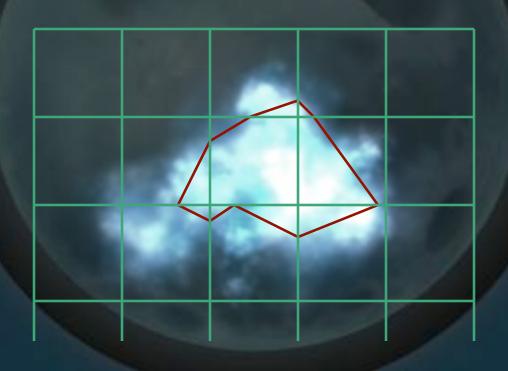
General idea: assign to current position



- Overview
  - Update velocities based on
    - Forces, then
    - Advection, then
    - Viscosity
  - Project velocities to zero divergence
  - Update densities based on
    - Input sources
    - Velocity
    - Diffusion (similar to viscosity, sometimes not used)
  - Draw it

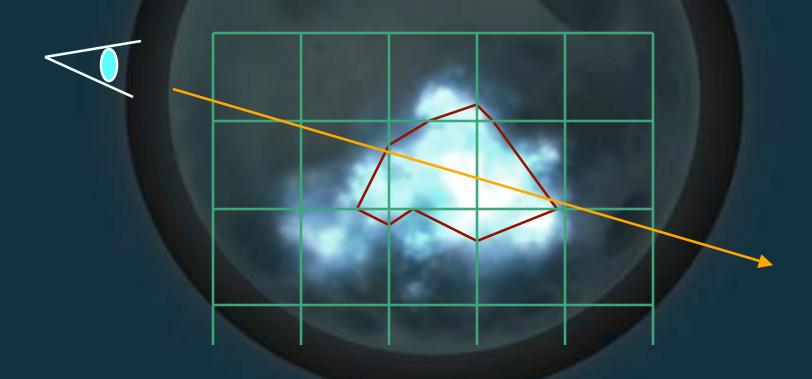
# Rendering Grid-Based

Build level surface



# Rendering Grid-Based

Determining color, transparency





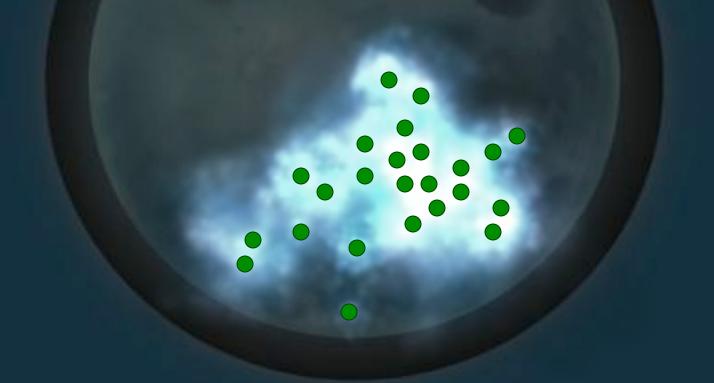
- Limited space
- Water "splashes" get lost
- Can be computationally expensive
- Dampens down
- But stable

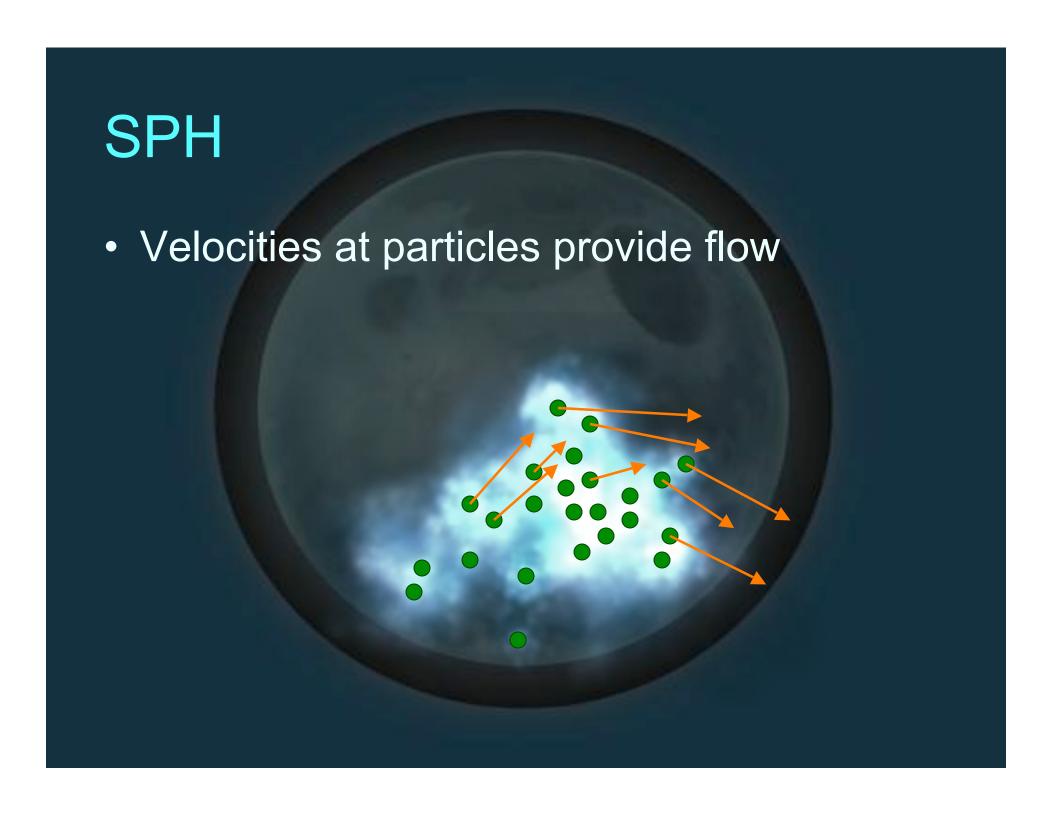
## Implementation

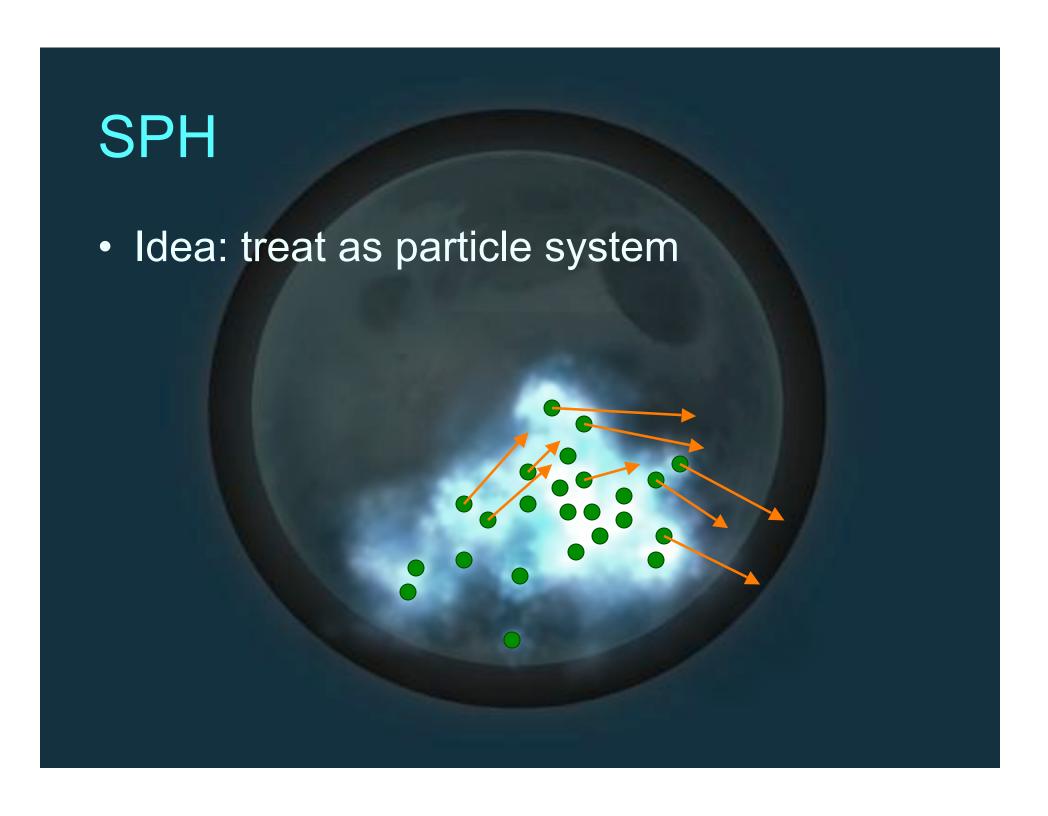
- Little Big Planet
  - "Death smoke"
  - Bubble pop
  - Other smoke effects
- Hellgate: London
- GDC09 NVIDIA demo

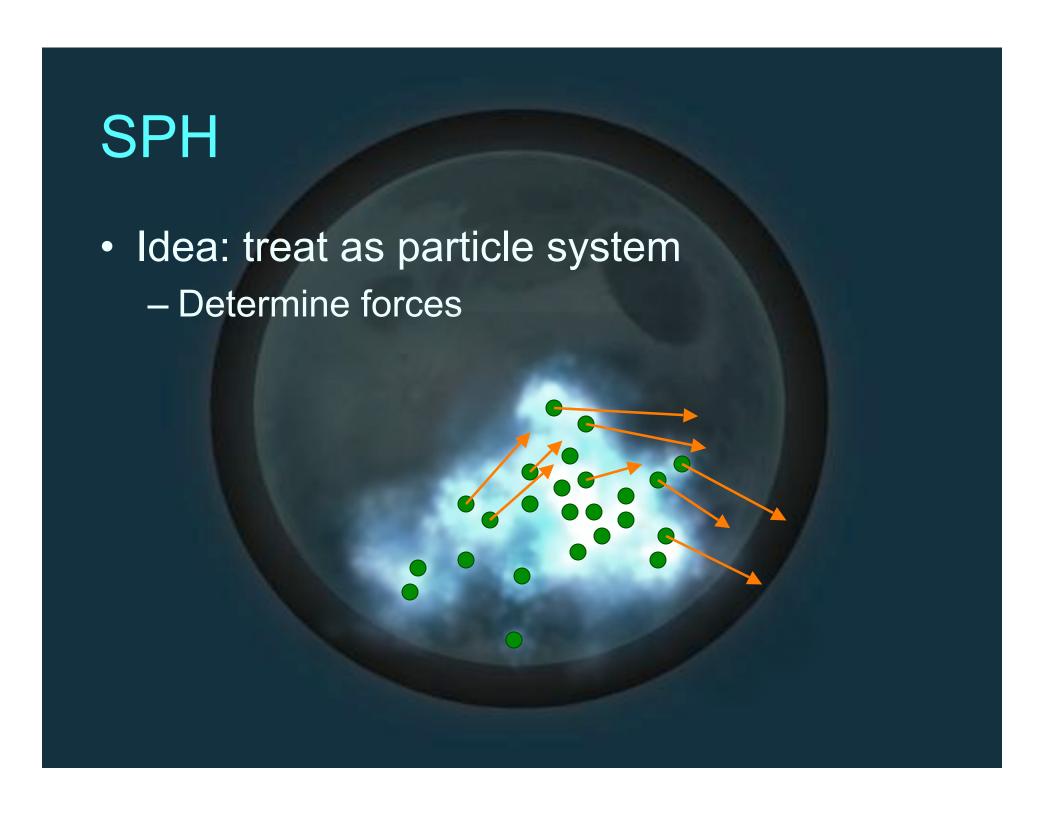
# Smoothed Particle Hydrodynamics

Approximate fluid with small(er) set of particles







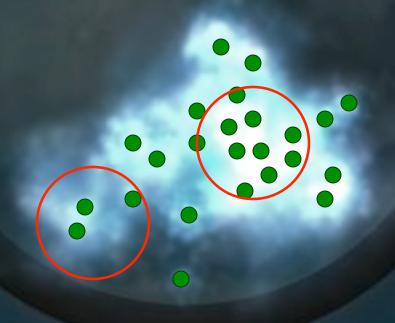




- Idea: treat as particle system
  - Determine forces
  - Update velocities, positions

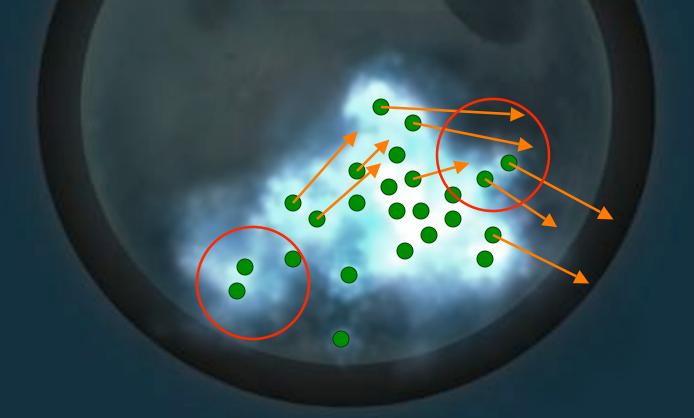


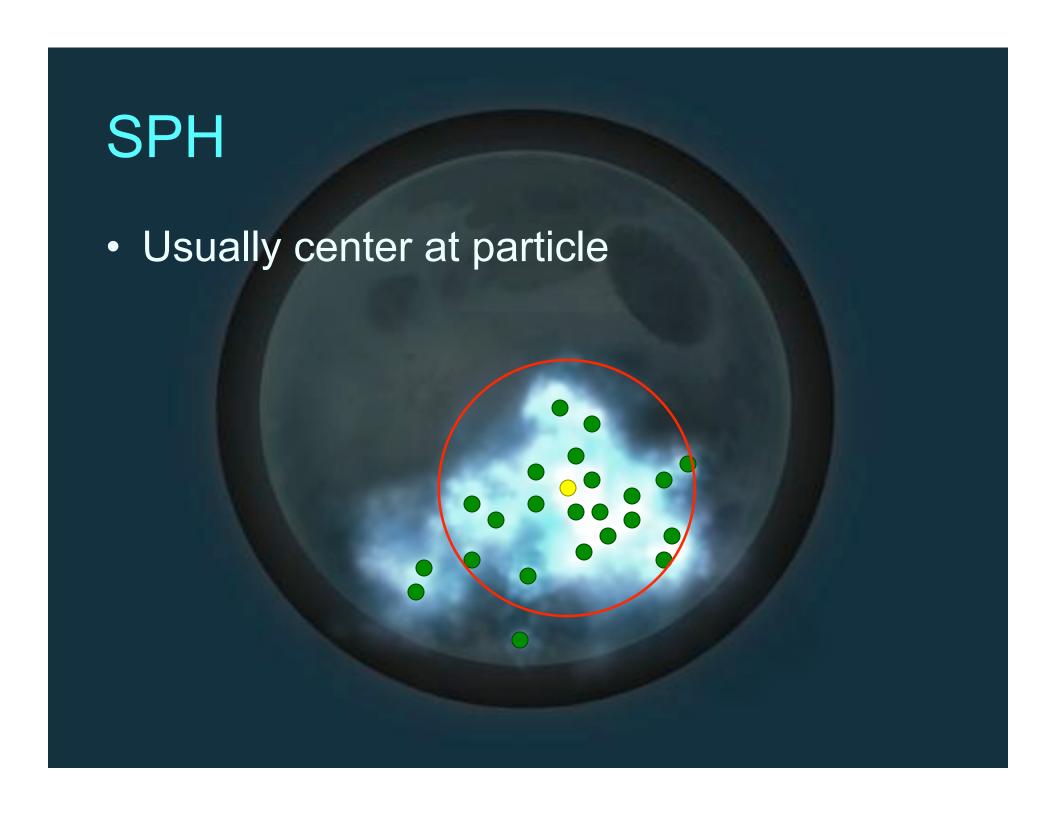
 Weighted average gives density (smoothing kernel)

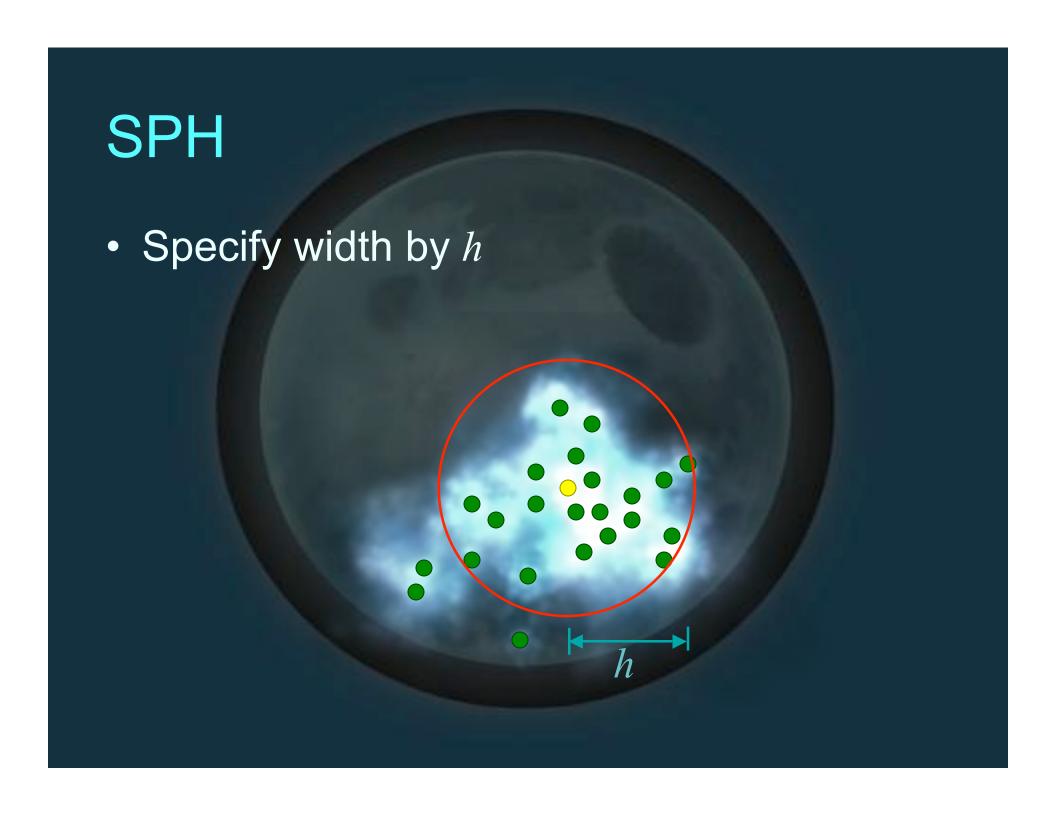




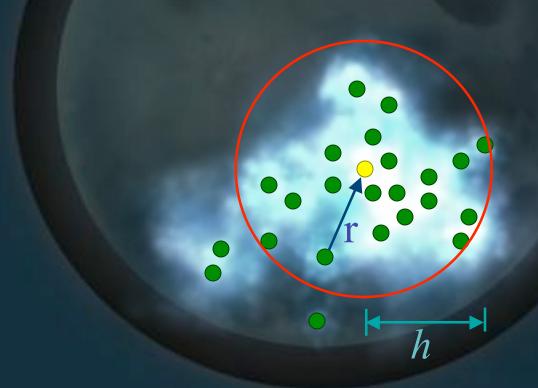
Can also use kernel to get general velocity

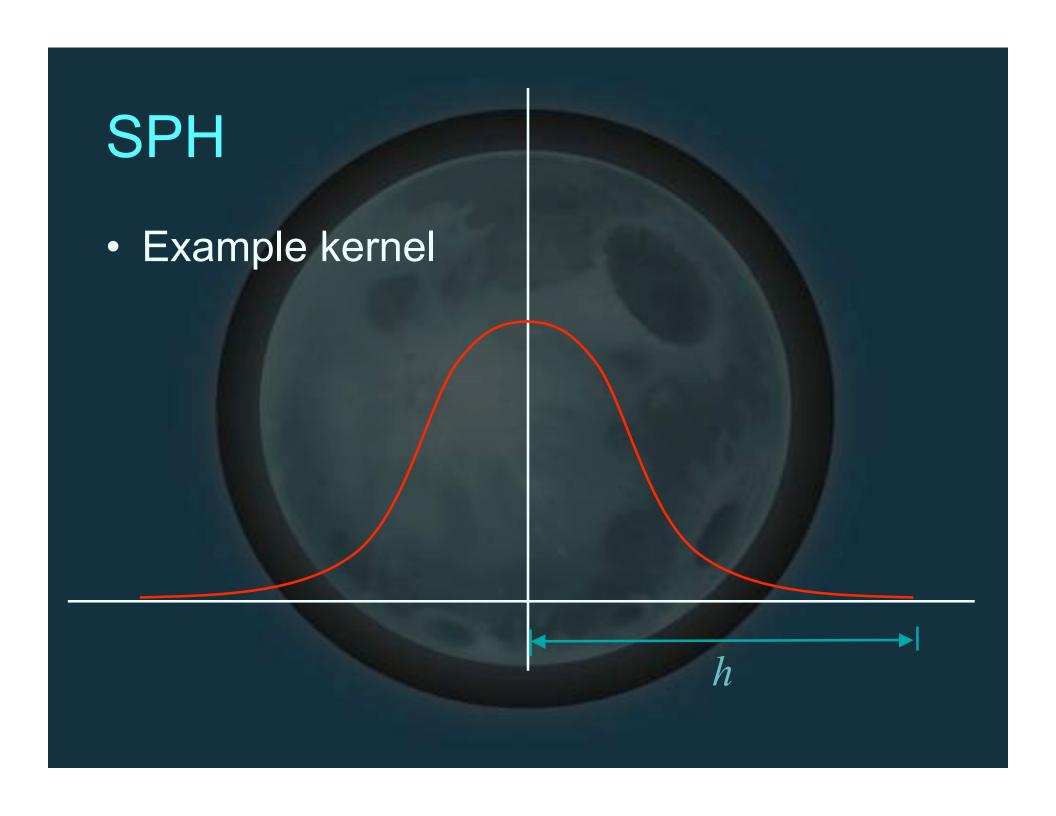






Specify vector from other particles by r





Common kernel

$$W_{\text{poly 6}}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

Common kernel

$$W_{\text{poly 6}}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

Clamps to zero at boundary

Common kernel

$$W_{\text{poly 6}}(\mathbf{r}, h) = \frac{315}{64\pi h^9} \begin{cases} (h^2 - r^2)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$

Clamps to zero at boundary Uses length squared

General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

General SPH rule

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Particle mass

General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \underbrace{A_{j}}_{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

Quantity at particle j

General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

Density at particle j

General SPH rule

$$A_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{A_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

Weighting function

Computing density

$$\rho_{S}(\mathbf{x}) = \sum_{j} m_{j} \frac{\rho_{j}}{\rho_{j}} W(\mathbf{r} - \mathbf{r}_{j}, h)$$

Computing density

$$\rho_{S}(\mathbf{x}) = \sum_{j} m_{j} W(\mathbf{r} - \mathbf{r}_{j}, h)$$



Local pressure

$$p_i = k\rho_i$$

k is gas constant

Local pressure

$$p_i = k\rho_i$$

k is gas constant

Can be unstable, so...

Local pressure (alternative)

$$p_i = k(\rho_i - \rho_0)$$

No effect on gradient, more stable

Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Back to Navier-Stokes

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Have fixed # particles and mass, so...

Back to Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Have fixed # particles and mass, so... mass is automatically conserved

Back to Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection automagically handled by particle update, so...

Back to Navier-Stokes

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Advection automagically handled by particle update, so...

Simplifies to

$$\frac{\partial \mathbf{u}}{\partial t} = -(\mathbf{u} \cdot \nabla)\mathbf{u} - \frac{1}{\rho}\nabla p + \nu \nabla^2 \mathbf{u} + \mathbf{f}$$

Simplifies to

$$\rho \frac{d\mathbf{v}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Change in velocity

Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = \nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Pressure

Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

Viscosity

Functional breakdown

$$\rho \frac{d\mathbf{u}}{dt} = -\nabla p + \mu \nabla^2 \mathbf{u} + \rho \mathbf{f}$$

**External forces** 

- Compute densities, local pressure
- Generate forces on particles
  - External
  - Pressure
  - Viscosity
- Update velocities, positions
- Render



Pressure

$$\mathbf{f}_{i}^{pressure} = -\nabla p(\mathbf{r}_{i})$$



Pressure

$$\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{j}}{\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

Pressure

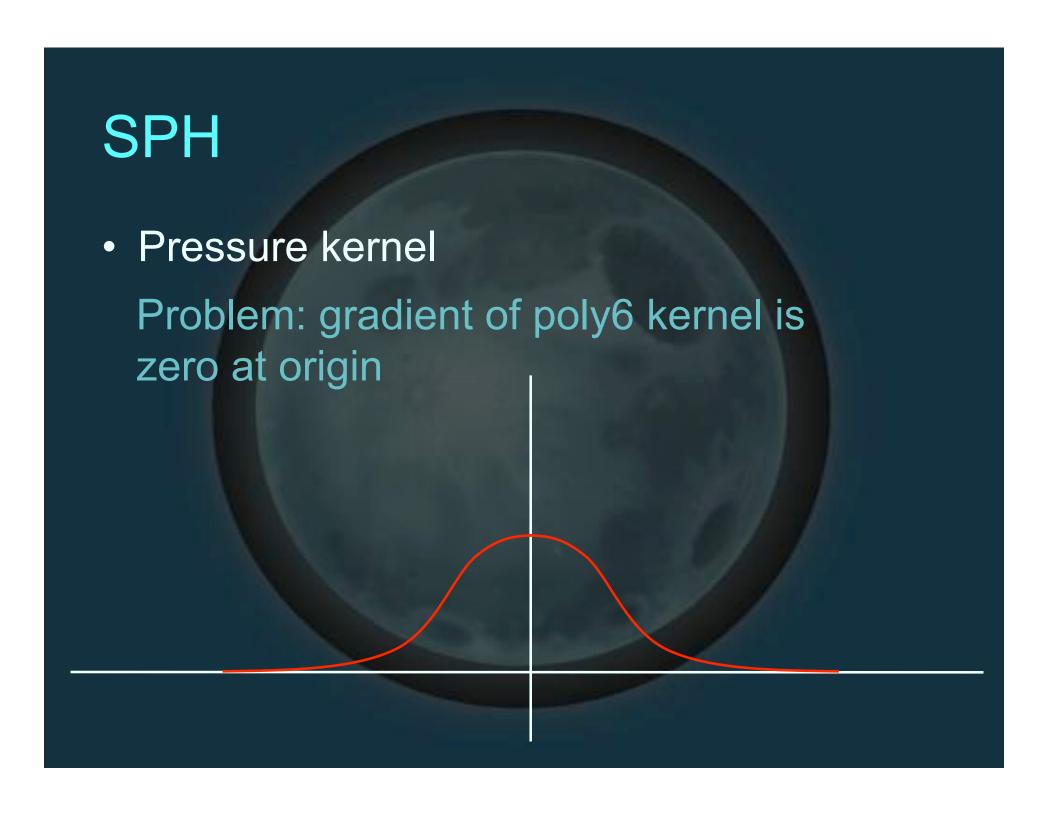
$$\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{j}}{\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

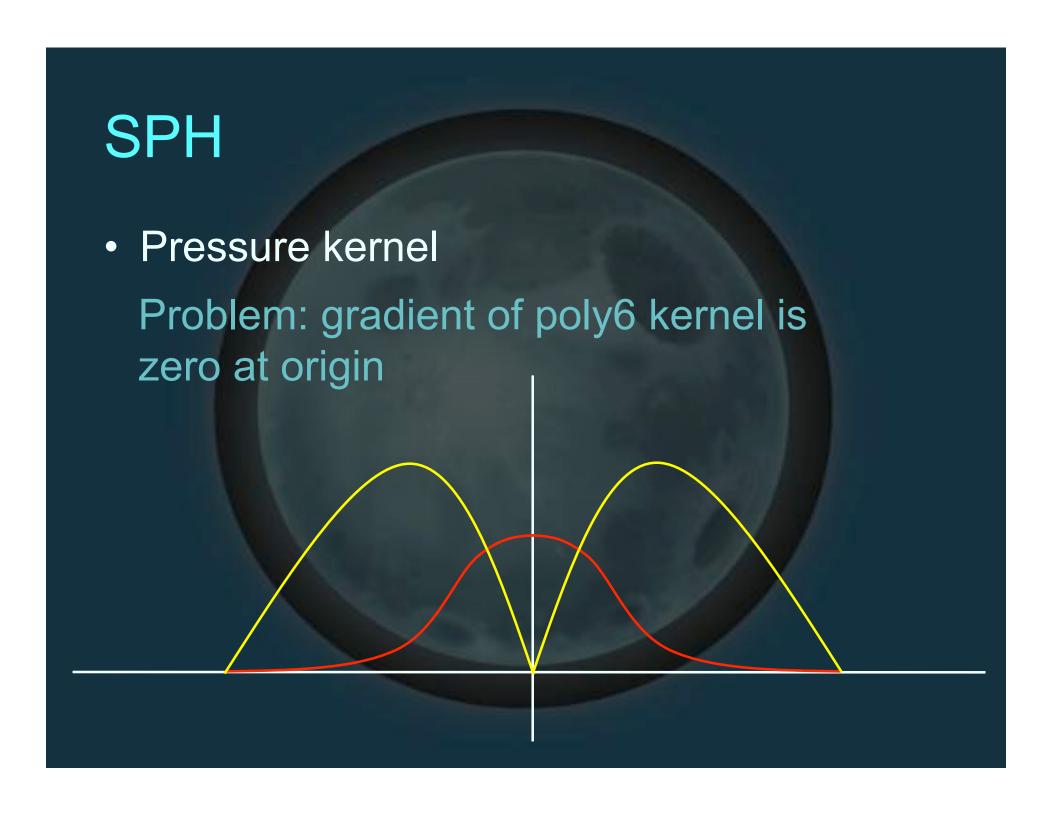
Asymmetric, so...

Pressure

$$\mathbf{f}_{i}^{pressure} = \sum_{j} m_{j} \frac{p_{i} + p_{j}}{2\rho_{j}} \nabla W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

Ensures 2-particle interaction equal





Pressure kernel

$$W_{\text{spiky}}(\mathbf{r},h) = \frac{15}{\pi h^6} \begin{cases} (h-r)^3 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$



Viscosity

$$\mathbf{f}_i^{\text{viscosity}} = \mu \nabla^2 \mathbf{v}(\mathbf{r}_i)$$



Viscosity

$$\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

Viscosity

$$\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

Also asymmetric, so...

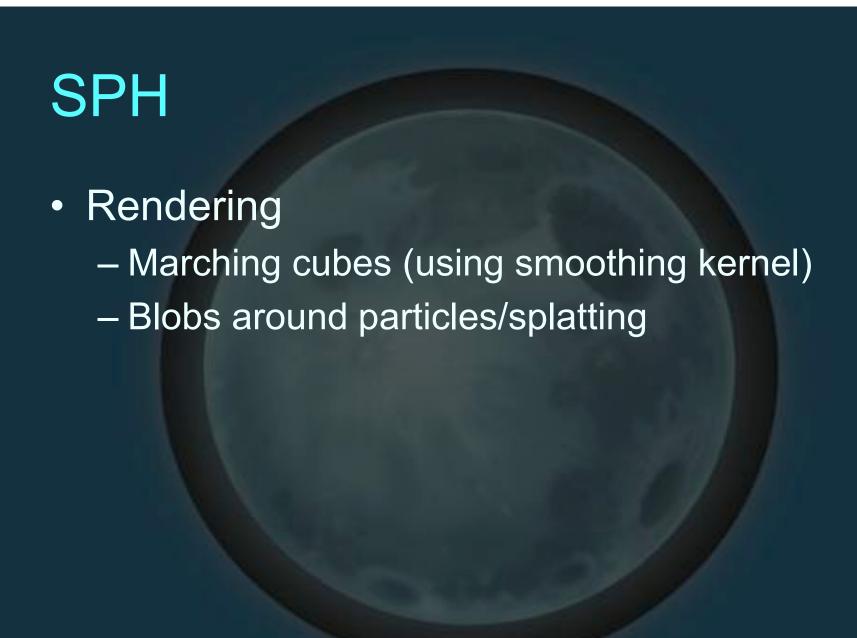
Viscosity

$$\mathbf{f}_{i}^{\text{viscosity}} = \mu \sum_{j} m_{j} \frac{\mathbf{v}_{j} - \mathbf{v}_{i}}{\rho_{j}} \nabla^{2} W(\mathbf{r}_{i} - \mathbf{r}_{j}, h)$$

Ensures 2-particle interaction opposite

Viscosity kernel

$$W_{\text{viscosity}}(\mathbf{r},h) = \frac{15}{2\pi h^3} \begin{cases} -\frac{r^3}{2h^3} + \frac{r^2}{h^2} + \frac{h}{2r} - 1 & 0 \le r \le h \\ 0 & \text{otherwise} \end{cases}$$



## SPH Implementations

- Takahiro Harada
- Kees van Kooten (Playlogic)
- NVIDIA PhysX
- Rama Hoetzlein\* (SPH Fluids 2.0)
- Takashi AMADA\*

\* Source code available

### **SPH Issues**

- Need a lot of particles
- Computing level surface can be a pain
- Can be difficult to get stable simulation

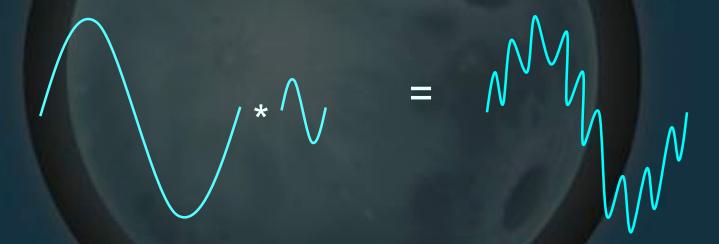
# **SPH Improvements**

- Spatial hashing
- Variable kernel width
- CFD/SPH Hybrid
  - CFD manages general flow
  - SPH "splashes"

#### **Surface Simulation**

- Idea: for water, all we care about is the airwater boundary (level surface)
- Why simulate the rest?
- This is what Insomniac R20 system does

- Done by Mike Day, based on Titanic water
  - Basic idea: convolve sinusoids procedurally

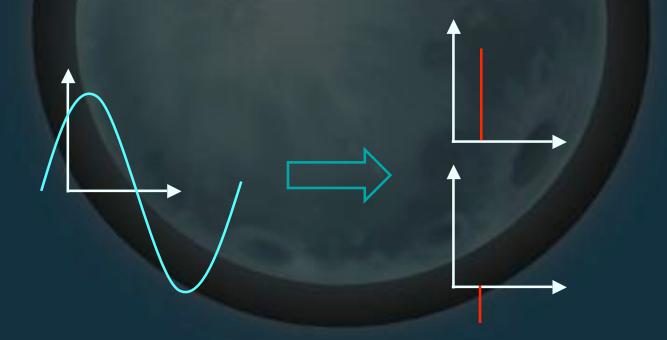


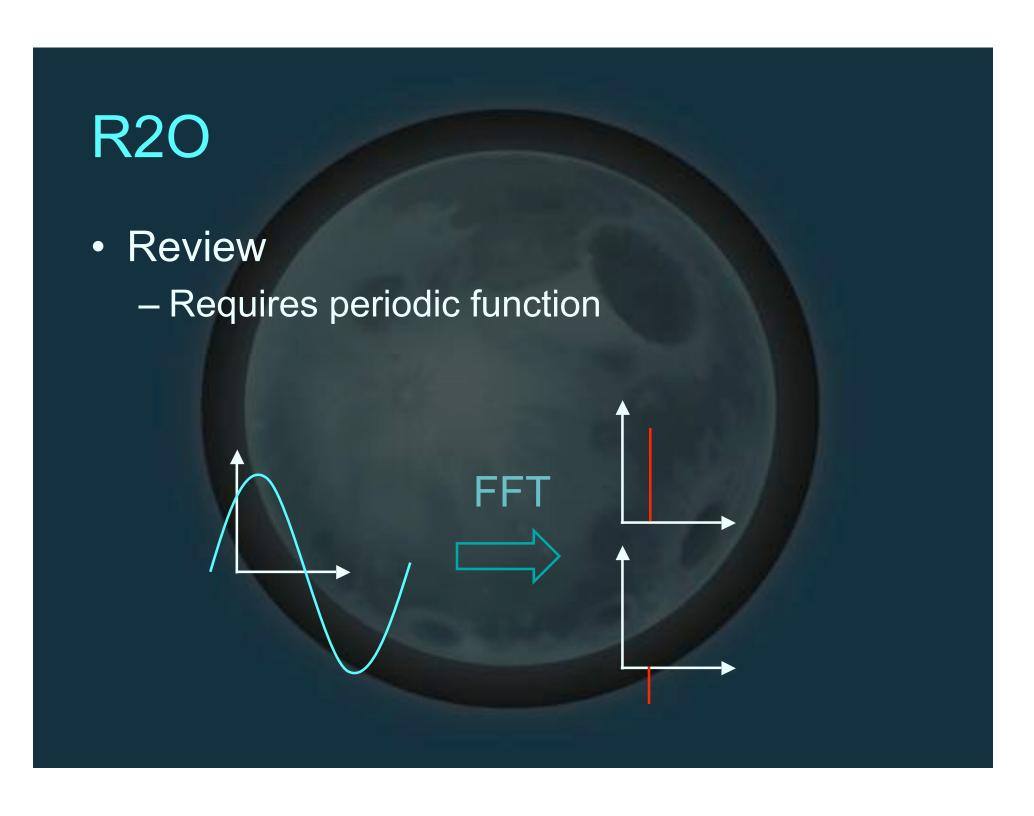
 Much cheaper to multiply in frequency domain and do FFT (assuming periodic)



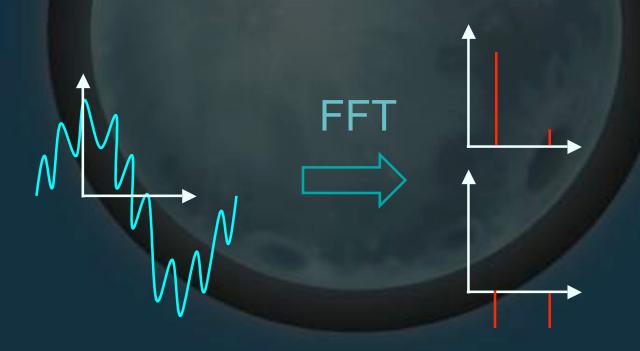


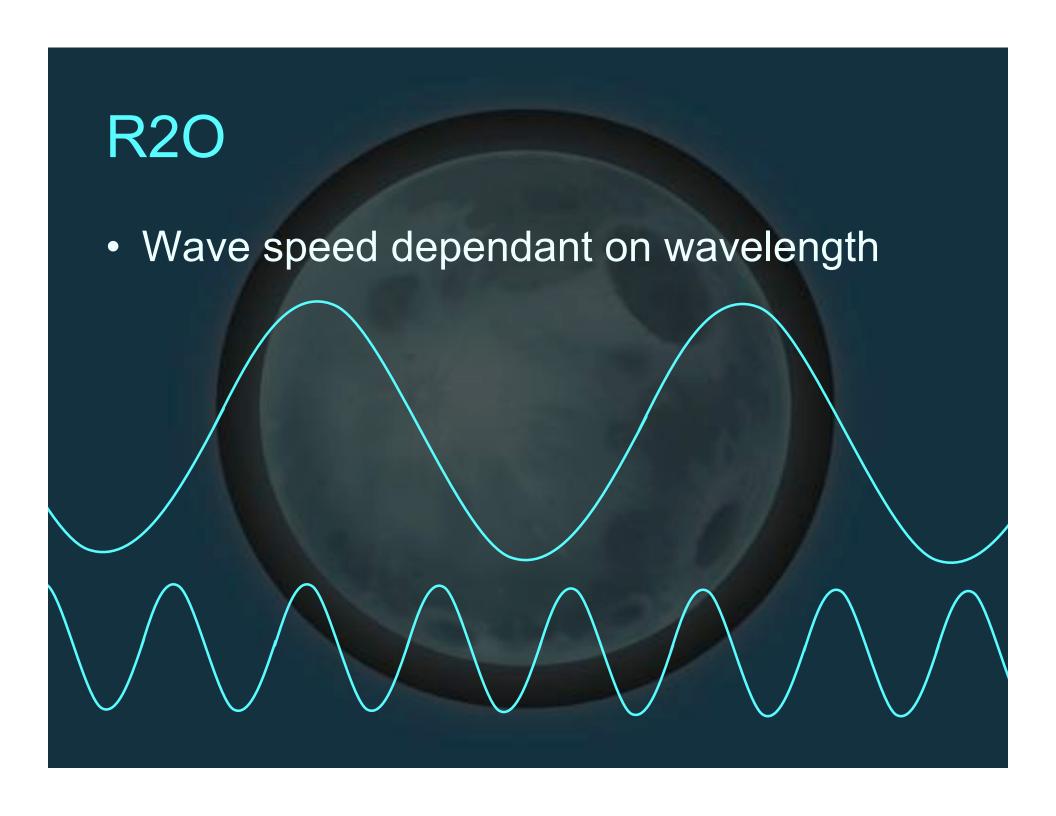
- Review
  - Can represent as magnitude+phase in frequency slot

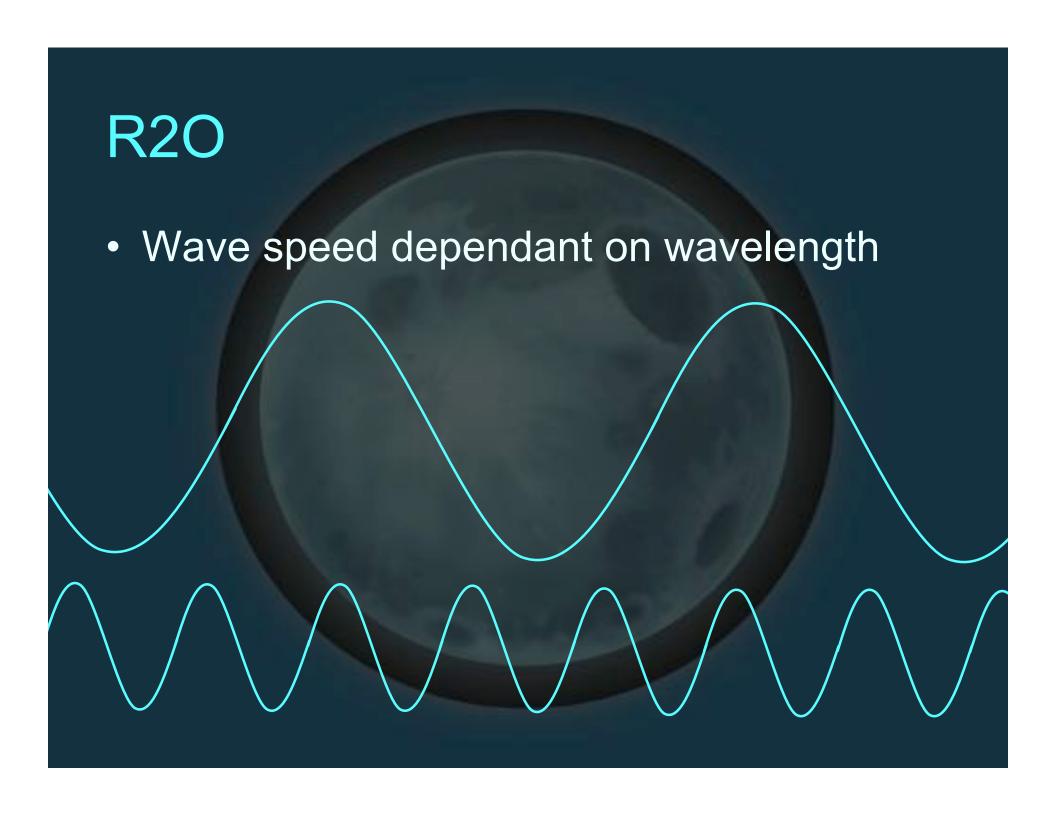


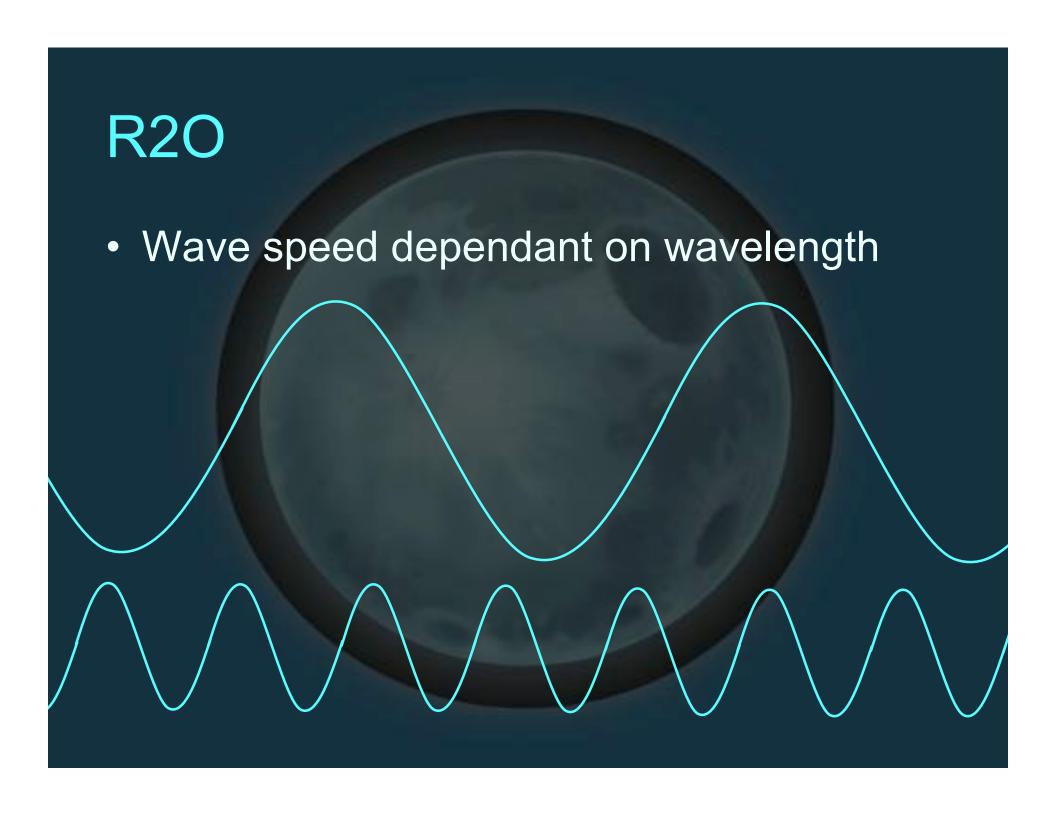


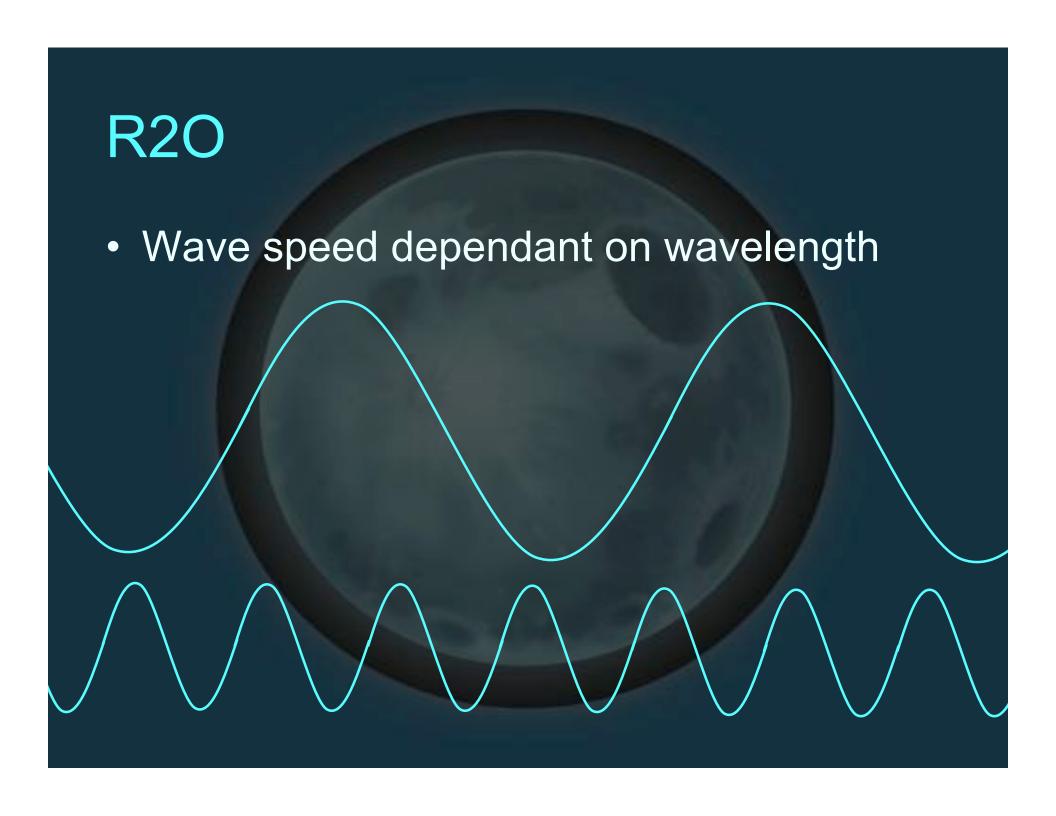
- Review
  - Multiple sinusoids end up at multiple entries













- Wave speed dependant on wavelength
  - Le. phases update at different rates
  - AKA dispersion

- General procedure
  - Start with convolved data in (r,φ) form
  - Update phase angles for each sinusoid
    - Angular velocity\*dt
    - Dependent on frequency
  - Do inverse FFT to get spatial result

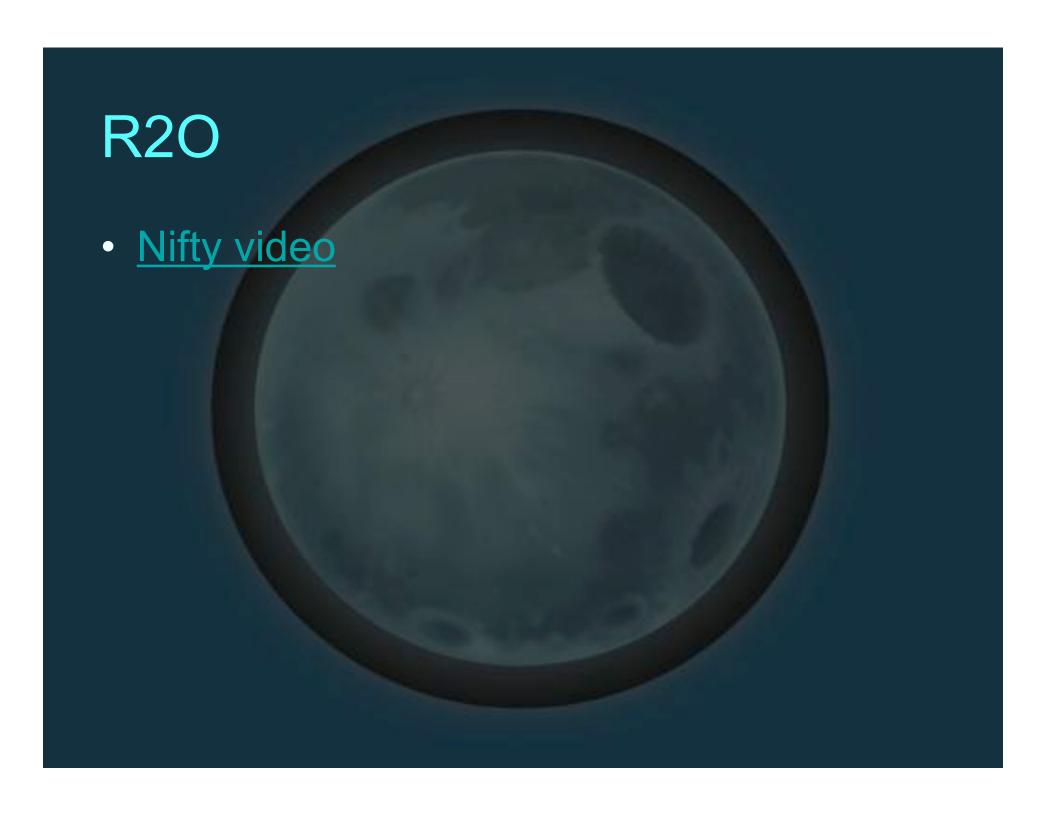


- FFT kernel limited to 32x32
- Combine multiple levels via LOD height field scheme
  - Gives high detail close to camera

- Interactive waves
  - Just adding in splashes looks fake
  - Instead, do some more FFT trickery so all our work occurs in the same domain
  - Non-periodic, so have to manage edges
  - Gives nice dispersion effects



- Rendering
  - Rendered as height field mesh
  - Add normal map for detail
  - Cube map/frame buffer map for reflections
  - Distortion effect for refractions



#### References

- Jos Stam, "Stable Fluids", SIGGRAPH 1999
- Mattias Müller, et. al, "Particle-Based Fluid Simulation for Interactive Applications", SIGGRAPH Symposium on Computer Animation 2003
- Jerry Tessendorf, "Simulating Ocean Water," SIGGRAPH Course Notes.
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